4. *Compton effect*

An incident photon with wave vector \( k_0 \) is scattered from a particle with rest mass \( m_0 \) and initial momentum \( p_0 = 0 \). The scattered photon has the wave vector \( k \).

Based on the conservation of the relativistic energy and momentum, determine the shift \( \lambda - \lambda_0 \) of the wave length of the scattered photon as a function of the scattering angle \( \theta \) between \( k \) and \( k_0 \).

(Remark: Special relativity yields the relation \( E^2 = c^2 p^2 + m^2 c^4 \) between the energy \( E \) and the momentum \( p \) of an object of rest mass \( m \).)

5. *Bohr model*

Consider the motion according to classical mechanics of an electron of charge \(-e\) in the electrostatic field of a proton of charge \(+e\).

(a) Determine the effective potential for the motion of the electron around the nucleus as a function of the radial distance.

(b) According to the Bohr model, electrons move on circular orbits and the angular momentum \( L \) can assume the values \( L = n\hbar, \; n \in \{1, 2, \ldots\} \). Determine the possible energies \( E_n \), orbital radii \( r_n \), and velocity ratios \( v_n/c \), where \( c \) is the speed of light.

(c) Bohr assumed that only radiation of frequency

\[
\omega(n_1, n_2) = \frac{E_{n_1} - E_{n_2}}{\hbar}, \quad n_1 > n_2
\]

(1)

can be emitted. Compare \( \omega(n + 1, n) \) for \( n \to \infty \) with the orbital frequency which results from classical mechanics using the results of part (b).

(d) For velocities \( v \ll c \), the power radiated by an accelerated charge is given by the classical expression

\[
P = \frac{e^2}{6\pi\varepsilon_0 c^3} \dot{v}^2
\]

(2)
in SI units. For the orbit corresponding to energy \( E_1 \), estimate the time during which the electron (considered to be a classical particle) would crash into the nucleus.
6. Fourier transform

Given a function \( f : \mathbb{R} \to \mathbb{C} \), its Fourier transform \( \tilde{f} \), provided it exists, is defined by

\[
\tilde{f}(q) := \int_{-\infty}^{\infty} \frac{dx}{\sqrt{2\pi}} f(x) \exp(-iqx).
\] (3)

The following relations hold:

\[
f(x) = \int_{-\infty}^{\infty} \frac{dq}{\sqrt{2\pi}} \tilde{f}(q) \exp(iqx) \quad \text{(inverse Fourier transform)} \] (4)

\[
\int_{-\infty}^{\infty} dx |f(x)|^2 = \int_{-\infty}^{\infty} dq |\tilde{f}(q)|^2 \quad \text{( Parseval’s theorem).} \] (5)

Consider the Gaussian

\[
f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x^2}{2\sigma^2}\right). \] (6)

Calculate the Fourier transform \( \tilde{f} \), plot \( f(x) \) and \( \tilde{f}(q) \), and check Eqns. (4) and (5).