# Subgraph Decomposition for Multi-Target Tracking
# – Supplementary Material –

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This supplement contains a proof of Lemma 1 in [1].

**Proof of Lemma 1** Lemma 1 in [1] states that subgraphs, and thus tracking results, which are maximally probable with respect to the probability measure we define are precisely the solutions of a particular instance of the Minimum Cost Subgraph Multicut Problem. Below, we prove this lemma.

PROOF Let $G = (V, E)$ be a graph and $Z \subseteq \{0,1\}^{V \cup E}$. For every $v \in V$, let $f^v \in \mathbb{R}^J$. For every $e \in E$, let $g^e \in \mathbb{R}^K$. Finally, let $\theta \in \mathbb{R}^J$ and $\theta' \in \mathbb{R}^K$. Moreover, recall from (10)–(13) in the main text the definition of the probability measure

$$\begin{aligned} & p(x, y, \theta, \theta' | f, g, Z) \\ \propto\ & p(Z|x, y) \cdot \prod_{v \in V} p(x_v | f^v, \theta) \cdot \prod_{j \in J} p(\theta_j) \\ & \cdot \prod_{e \in E} p(y_e | g^e, \theta') \cdot \prod_{k \in K} p(\theta'_k) \end{aligned} \tag{i}$$

with

$$p(Z|x,y) \propto \begin{cases} 1 & \text{if } (x,y) \in Z \\ 0 & \text{otherwise} \end{cases} \tag{ii}$$

$$p(x_v = 1 | f^v, \theta) = \frac{1}{1 + \exp(-\langle \theta, f^v \rangle)} \tag{iii}$$

$$p(y_e = 1 | g^e, \theta') = \frac{1}{1 + \exp(-\langle \theta', g^e \rangle)} \tag{iv}$$

$$p(\theta_j) = \mathcal{N}(0, \sigma^2) \tag{v}$$

$$p(\theta'_k) = \mathcal{N}(0, \rho^2) \ . \tag{vi}$$

Although $p(Z|x, y)$ can be zero, the probability measure is well-defined for any $Z \neq \emptyset$ because $p(x_v|f^v, \theta) > 0$, by (iii), and $p(y_e|g^e, \theta') > 0$, by (iv).

It is such that

$$\begin{aligned} & p(x, y | \theta, \theta', f, g, Z) \\ =\ & p(Z|x, y) \prod_{v \in V} p(x_v | f^v, \theta) \prod_{e \in E} p(y_e | g^e, \theta') \end{aligned} \tag{vii}$$

because $p(\theta_j) > 0$, by (v), and $p(\theta'_k) > 0$, by (vi), and by conditioning on $\theta$ and $\theta'$.

Moreover, it is such that

$$\begin{aligned} & \underset{\substack{x \in \{0,1\}^V \\ y \in \{0,1\}^E}}{\operatorname{argmax}} \ p(x, y | \theta, \theta', f, g, Z) \\ =\ & \underset{(x,y) \in Z}{\operatorname{argmax}} \ \prod_{v \in V} p(x_v | f^v, \theta) \prod_{e \in E} p(y_e | g^e, \theta') && \text{(viii)} \\ =\ & \underset{(x,y) \in Z}{\operatorname{argmax}} \ \sum_{v \in V} \log p(x_v | f^v, \theta) + \sum_{e \in E} \log p(y_e | g^e, \theta') && \text{(ix)} \\ =\ & \underset{(x,y) \in Z}{\operatorname{argmax}} \ \sum_{v \in V} \langle \theta, f^v \rangle x_v + \sum_{e \in E} \langle \theta', g^e \rangle y_e \ . && \text{(x)} \end{aligned}$$

In this statement that concludes the proof, (viii) holds by (vii) and (ii). (ix) follows by the strict monotonicity of the logarithmic function, and (x) follows by the arithmetic transformations stated below for $p(x_v|f^v, \theta)$. The transformations for $p(y_e|g^e, \theta')$ are analogous.

$$\begin{aligned} & \log p(x_v | f^v, \theta) \\ & = x_v \log p(x_v = 1 | f^v, \theta) + (1 - x_v) \log p(x_v = 0 | f^v, \theta) \\ & = x_v \log \frac{p(x_v = 1 | f^v, \theta)}{p(x_v = 0 | f^v, \theta)} + \log p(x_v = 0 | f^v, \theta) \\ & = x_v \langle f^v, \theta \rangle - \log(1 + \exp(-\langle f^v, \theta \rangle)) \end{aligned} \tag{xi}$$

Here, (xi) follows by (iii). Note that the second term in (xi) does not depend on $x_v$ and can hence be dropped from the objective function (x).

## References

[1] S. Tang, B. Andres, M. Andriluka, and B. Schiele. Subgraph decomposition for multi-object tracking. In *CVPR*, 2015. 1